***Why do we need different kinds of averages?***

The average that we're used to is found by adding all the values in a data set, and then dividing the sum by the number of values in that data set; but this average might be misleading.

[Mean, Median, Mode, and Range](https://www.mathhelp.com/how_to/central_tendency/mean/?id=1162&utm_campaign=purplemath&utm_source=meanmode.htm&utm_medium=banner)

A typical example would be the case where nearly every person in a given population lives on about two dollars a day, but there is a small elite with incomes in the millions. The numerical average can mislead by suggesting that the average (in this case, we mean "typical") person earns a few tens of thousands per year. But this does not accurately reflect what we mean when we're trying to discuss the "average" income. This is why the average income is typically expressed by a different sort of average.

***What are the three averages?***

The three averages are:

* The "mean" is the "average" you're used to, where you add up all the numbers and then divide by the number of numbers.
* The "median" is the "middle" value in the list of numbers. To find the median, your numbers have to be listed in numerical order from smallest to largest, so you may have to rewrite your list before you can find the median.
* The "mode" is the value that occurs most often. If no number in the list is repeated, then there is no mode for the list.

The "range" of a list a numbers is just the difference between the largest and smallest values. It expresses "spread", being how far the values are distributed (or how concentrated they are).

* Find the mean, median, mode, and range for the following list of values:

13, 18, 13, 14, 13, 16, 14, 21, 13

The mean is the usual average, so I'll add and then divide:

(13 + 18 + 13 + 14 + 13 + 16 + 14 + 21 + 13) ÷ 9 = 15

Note that the mean, in this case, isn't a value from the original list. This is a common result. You should not assume that your mean will be one of your original numbers; you should not be surprised when it isn't.

The median is the middle value, so first I'll have to rewrite the list in numerical order:

13, 13, 13, 13, 14, 14, 16, 18, 21

There are nine numbers in the list, so the middle one will be the (9 + 1) ÷ 2 = 10 ÷ 2 = 5th number:

13, 13, 13, 13, 14, 14, 16, 18, 21

So the median is 14.

The mode is the number that is repeated more often than any other, so 13, I see from my listing above, is the mode.

The largest value in the list is 21, and the smallest is 13, so the range is 21 − 13 = 8.

mean: 15  
median: 14  
mode: 13  
range: 8

Note: The formula for the place to find the median is "([the number of data points] + 1) ÷ 2", but you don't have to use this formula. You can just count in from both ends of the list until you meet in the middle, if you prefer, especially if your list is short. Either way will work.

* **Find the mean, median, mode, and range for the following list of values:**

1, 2, 4, 7

The mean is the usual average:

(1 + 2 + 4 + 7) ÷ 4 = 14 ÷ 4 = 3.5

The median is the middle number. In this example, the numbers are already listed in numerical order, so I don't have to rewrite the list. But there is no "middle" number, because there are an even number of numbers.

Because of this, the median of the list will be the mean (that is, the usual average) of the middle two values within the list. The middle two numbers are 2 and 4, so:

(2 + 4) ÷ 2 = 6 ÷ 2 = 3

So the median of this list is 3, a value that isn't in the list at all.

The mode is the number that is repeated most often, but all the numbers in this list appear only once, so there is no mode.

The largest value in the list is 7, the smallest is 1, and their difference is 6, so the range is 6.

mean: 3.5  
median: 3  
mode: none  
range: 6

The values in the list above were all whole numbers, but the mean of the list was a decimal value. Getting a decimal value for the mean (or for the median, if you have an even number of data points) is perfectly okay; don't round your answers to try to match the format of the other numbers.

* **Find the mean, median, mode, and range for the following list of values:**

8, 9, 10, 10, 10, 11, 11, 11, 12, 13

The mean is the usual average, so I'll add up and then divide:

(8 + 9 + 10 + 10 + 10 + 11 + 11 + 11 + 12 + 13) ÷ 10 = 105 ÷ 10 = 10.5

The median is the middle value. In a list of ten values, that will be the (10 + 1) ÷ 2 = 5.5-th value; the formula is reminding me, with that "point-five", that I'll need to average the fifth and sixth numbers to find the median. The fifth and sixth numbers are the last 10 and the first 11, so:

(10 + 11) ÷ 2 = 21 ÷ 2 = 10.5

The mode is the number repeated most often. This list has two values that are repeated three times; namely, 10 and 11, each repeated three times.

The largest value is 13 and the smallest is 8, so the range is 13 − 8 = 5.

mean: 10.5  
median: 10.5  
modes: 10 and 11  
range: 5